BASIC FORMULAS

$\frac{40}{625}$ $\frac{8}{105}$ 6 4 24

$x^3 - y^3 = (x-y)(x^2+xy+y^2)$

$x^3 - 4x^2$

$x^3 + px + q = 0$

6 | 2 −5 −46 +24 → $(x-6)(2x^2+7x-4)$ → $(2x-1)(x+4)$

$x = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$ ±40

quadratic: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

−8 | 1 8 −3 −24 → $x^3 + b = (x+\sqrt[3]{b})(x^2 + \sqrt[3]{b}x + \sqrt[3]{b^2})$ $x^3 - 1 = (x-1)(x^2+x+1)$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$ $a^3 + b^3 = (a+b)(a^2-ab+b^2)$ $x - c$ ⇒ c is factor and x int

4 | 6 −16 25 +18 8 → 24 20 −20 8 → 6 5 −5 −2 0

$\frac{3x^2+4x-1}{x-1}$ → 1 | 3 4 −1 → 3 7 6 remain

STAT FUNC — $y = |x|$ — INT( )

$-\frac{1}{3}$ | 5 −5 −2 → −2 −1 +2 → 6 3 −6 0

$6x^2+3x-6$ → $3(2x^2+x-2)$ → $3(2x-1)(x+2)$

$x-2$ | x^3+0x^2+0x-8 → 2 | 1 0 0 −8 → x^2+2x+4

USE b^2-4ac discriminate 2 4 OR $= (x+2)^2$

−3 | 1 −8 0 −1 → −3 33 −99 → 1 −11 33 −100

1 | 1 0 2 4 1 → 1 3 → 1 1 3 7 8

6 0 a 6 61

remainder of $\frac{P(x)}{x-c} = P(c)$ binomial $r_1 + r_2 = \frac{-b}{a}$

trin $r_1+r_2 = \frac{-b}{a}$ $r_1r_2 + r_2r_3 + r_3r_1 = \frac{c}{a}$ $r_1r_2r_3 = -\frac{d}{a}$ $r_1r_2 = \frac{c}{a}$

2 a a

$ax^2+bx+c=0$ roots are x-int c = y int

at a time = 2nd coef, 2 at time =

$c^2 = a^2 + b^2 - 2ab\cos C$

$r_1+r_2+r_3 = \frac{-b}{a}$ $r_1r_2 + r_2r_3 + r_3r_1$ ...

$|a+bi| = \sqrt{a^2+b^2}$

$\frac{3}{2}$ | 10 −25 | $\frac{3}{2}$ 6

1 $\frac{3}{2}$ 4 $\frac{3}{6}$

$\frac{3}{2}$ 4

R. ROOT

RATIONAL ROOT

RATIONAL ROOT TEST — if is u root.

Q | lead coef, P | constant ≠ 0

$x^3-6x^2+11x-6=0$

P: ±1 −6

q: 1 1

roots = {±1, ±2, ±3, ±6}

IF P/Q is by + # and all q coefficient is non-neg, reached upper bound

IF P/Q is by − # and all q coefficient alternates + − + −, reached lower bound

$\sin(A-B)=\sin A\cos B-\cos A\sin B$

$\cos(A-B)=\cos A\cos B+\sin A\sin B$

$\cos(A+B)=\cos A\cos B-\sin A\sin B$

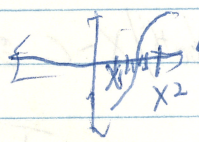
$\sin(A+B)=\sin A\cos B+\cos A\sin B$

$\sin\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{2}}$

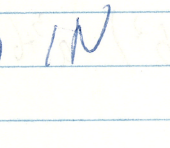
$\cos\frac{\theta}{2}=\pm\sqrt{\frac{1+\cos\theta}{2}}$

$\tan\frac{\theta}{2}=\frac{\sin\theta}{1+\cos\theta}$

Location Theorum



∈ ZERO IN



$x^3-2x^2+x-1=0$

+ − + −

3 + reals

1 + reals

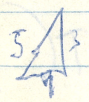
$4x^3-8x^2+25x+420$ neg

+ − + +

2 + reals

0 reals

1 real

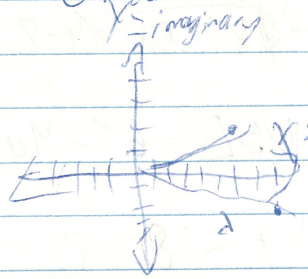


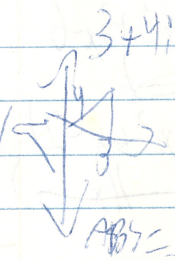
$6+2i=x+(2y)i$

$6=x$

$2y=2$

$6+2i$ ∠ $8+2i$





ABS(3+4i) = 5

ABSOLUTE VALUE's Distance from 0

$a+bi=r\cos\theta+i\,r\sin\theta$

$z=\frac{\sqrt{3}}{2}+\frac{1}{2}i$

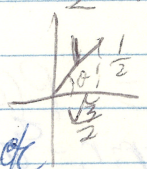
$r=1\quad\theta=\frac{\pi}{6}$

$z=\sqrt{2}-\sqrt{2}i=2\left(\cos\frac{\pi}{4}+i\sin\frac{7\pi}{4}\right)$

$=3i$

$\frac{\sqrt{3}}{2}+\frac{1}{2}i$

$-\sqrt{3}\ \frac{1}{2}\ +\frac{1}{4}$



$z_1=r_1(\cos\theta_1+i\sin\theta_1)\quad z_2=$

$z_1z_2=(r_1(\cos\theta_1+i\sin\theta_1))(r_2(\cos\theta_2+i\sin\theta_2))$

$=r_1r_2(\cos\theta_1\cos\theta_2+i\cos\theta_1\sin\theta_2+i\sin\theta_1\cos\theta_2+i^2\sin\theta_1\sin\theta_2)$

$r_1r_2(\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2)+i(\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2)$

$r_1r_2(\cos(\theta_1+\theta_2)+i(\sin(\theta_1+\theta_2))$

$\frac{z_1}{z_2}=\frac{r_1}{r_2}(\cos(\theta_1-\theta_2)+i(\sin(\theta_1-\theta_2))$

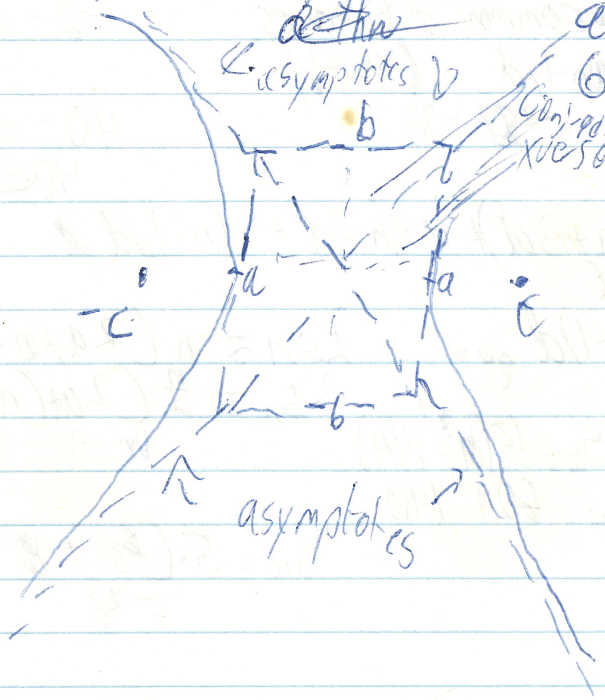
50000 ~~000~~
~~7000~~

Hyperbola

x and y axis inversely $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ hyperbola conic

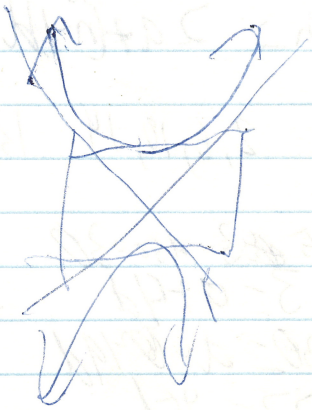
a = transverse axis c = foci

b = conjugate axis $c^2 = a^2 + b^2$



$\text{eccentricity} = \frac{\pm ec}{a \pm bx}$

$\text{asymptotes} = y = \frac{b}{a}x$



$\frac{y^2}{a^2} - \frac{x^2}{b^2}$

asymp $y = \frac{\pm a}{b}$

PARABOLA

LINE POINT EQUIDISTANT

$y = \frac{-b}{2a}$ axis of symmetry

$y = ax^2 + bx + c$

$3x - 4y = 12$ $3x - 4y - 12 = y$

$3x - 3y - 12$

$x^2 - 8x + 9 = 0$ $x^2 - 2x + 16 + y^2 - 6y + 9 = 9 + 9 + 16$

$(x-9)(x+1) = 0$

$x = 9$ $x = -1$

SEQUENCES

Fibonacci sequences: 1, 1, 2, 3, 5, 8, 13 (add prev. 2) recursive

Sigma $\sum_{k=x_5}^{n} k a$

k => dummy variable

x thru n of ka; sum of all #'s

a => formula w/ n,

$\sum_{j=1}^{n} (a_{j+1} - a_j)$ collapsing (telescoping) series,

$(a_2 - a_1) + a_3 - a_2 + a_4 - a_3 + a_5 - a_4 = a_5 - a_1$

CS1022
26/1/44

ARITHMETIC PROGRESSIONS sequences that are made by common differences.

nth term → $a_n = a_1 + (n-1)d$ (OUT!)

$40 = a$

$2 = a + 6d$

$-a - 2$

$40 = a + (-a-2)d$

$40 = a - ad - 2d$

$\begin{cases} S_n = a + (a+d) + (a+2d) + (a+3d) + \ldots + a + (n-2)d + a(n-1)d \\ S_n = a + (n-1)d + a + (n-2)d + \ldots \\ S_n = 2a + (n-1)d + 2a + (n-1)d \end{cases}$

$2S_n = n(2a + (n-1)d)$

$S_n = \frac{n}{2}(2a + (n-1)d)$

Or $S_n = \frac{n}{2}(a + a_n)$

2, 6, 10, 14 1st 30 $S_{30} = 15(4 + 144)$

$S_{30} = 60 + 1460$

$a = 5$ $a_{50} = 30$ $S_{10} = ?$

$3 = a + 4d$

$30 = a + 49d$

$27 = 45d$

$3 = 5d$ $d = \frac{3}{5}$

$\frac{15}{5} = a + \frac{12}{5}$

$\frac{3}{5} = a$

$S_{10} = 5(5 + \frac{27}{5})$

$5\frac{27}{5}$ 33

GEOMETRIC RATIO common RATIO 1, 2, 4, 8, 16 ... $\frac{2}{1} = \frac{4}{2} = \frac{8}{4} = \frac{16}{8}$ $a_n = ar^{n-1}$

$a_1 = a$ $a_3 = 4$ $a_2 = x$

$729 = 458$

$\frac{x}{2} = \frac{4}{x}$ $x^2 = 36$ $x = 6$

$64r^6 = 2000$

$16r^6 = 2000$

$4r^3 = 500$

$r^3 = 125 = 5$

$\frac{23}{1024}$

$S_n = a + ar + ar^2 \ldots ar^{n-1}$

$rS_n = ar + ar^2 + ar^3 \ldots ar^n$

$S - rS_n = (a - ar^n)$

$S_n(1-r) = a(1-r^n)$

$S_n = \frac{a(1-r^n)}{1-r}$

$\frac{4}{5}$

$\frac{1}{2} \cdot \frac{1023}{1024} = \frac{1023}{1024}$

$512 < 3584$

$n = \infty$ $\frac{1}{2^n} = 0$ n cancels out

for final $S_\infty = \frac{a}{1-r} = 1$

$\frac{3}{3}/\frac{3}{3} = 1\frac{2}{5}$

$\frac{3}{3}/\frac{4}{3} = \frac{1}{2}$

$\frac{4(1-1024)}{3}$ $\frac{4092}{3}$

BINOMIAL EXPANSION $(a+b)^x =$ $a^{x-(n-1)}b^{n-1}$

nth term

PASCAL'S TRI $10(8+5) = 2800$ 1364

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1